Conserved mass aggregation model with mass-dependent fragmentation

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We study a conserved mass aggregation model with mass-dependent fragmentation in one dimension. In the model, the whole mass m of a site isotropically diffuse with unit rate. With rate ω, a mass m^λ is fragmented from the site and moves to a randomly selected nearest neighbor site. Since the fragmented mass is smaller than the whole mass m of a site for $\lambda < 1$, the on-site attractive interaction exists for the case. For $\lambda = 0$, the model is known to undergo the condensation phase transitions from a fluid phase into a condensed phase as the density of total masses (ρ) increases beyond a critical density ρ_c. For $0 < \lambda < 1$, we numerically confirm for several values of ω that ρ_c diverges with the system size L. Hence in thermodynamic limit, the condensed phase disappears and no transitions take place in one dimension. We also explain that there are no transitions in any dimensions.

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Nonequilibrium phase transitions have been studied extensively during last decades and observed in various systems [1, 2, 3, 4]. As in equilibrium, nonequilibrium transitions can be classified into universality classes according to conservation laws and symmetries, which are characterized by several critical exponents.

In this paper, we study condensation phase transitions. Nonequilibrium condensation phase transitions from fluid phase into condensed phase have been observed in a variety of phenomena ranging from traffic flow to polymer gels [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In the steady state, a finite fraction of total particles condenses on a single site in condensed phase when the total particle density ρ is increased beyond a certain critical value ρ_c. In fluid phase below ρ_c, the particle number of each site fluctuates around ρ without the condensation [5, 15].

The simplest model exhibiting the condensation is zero-range process (ZRP) [5]. In ZRP, many identical particles occupy sites on a lattice. Each site may contain an integer number of particles and one of these particles can hop to one of the nearest neighboring sites with a rate that depends on the number of particles on the site of departure. The chipping (single-particle dissociation) and aggregation processes of ZRP describe various condensations such as traffic jam [6], bunching of buses [7], coalescence of shaken steel balls [8] and the condensation on complex networks [9].

Another important class of condensation transitions emerges when the diffusion of the whole particles of a site is allowed in addition to the chipping and aggregation. These processes arise in a variety of phenomena such as polymer gels [10], the formation of colloidal suspensions [11], river networks [12, 13] and cloud formation [14]. Recently studied symmetric conserved-mass aggregation(SCA) model is the simplest one incorporating diffusion, chipping and aggregation upon contact [15, 16]. In one dimensional SCA model, the mass m_i of site i moves either to site $i-1$ or to site $i+1$ with the unit rate, and then $m_i \to 0$ and $m_{i\pm1} \to m_{i\pm1} + m_i$. With the rate ω, unit mass chips off from site i and moves to one of the nearest neighboring sites; $m_i \to m_i - 1$ and $m_{i\pm1} \to m_{i\pm1} + 1$. As total masses are conserved, the conserved density ρ and ω determine the phase of SCA model. The condensation transition arises from the competition between diffusion and chipping processes. The diffusion of masses tends to produce massive aggregates and consequently creates more vacant sites. The chipping of unit mass tends to prevent the formation of aggregates, so that it leads to a replenishment of the lower end of mass distribution.

The single site mass distribution $P(m)$, i.e., the probability that a site has mass m in the steady state, was shown to undergo phase transitions on regular lattices [15]. For a fixed ω, as ρ is varied across the critical value $\rho_c(\omega)$, the behavior of $P(m)$ for large m was found to be [15]

$$P(m) \sim \begin{cases} e^{-m/m^*} & \rho < \rho_c(\omega) \\ m^{-\tau} & \rho = \rho_c(\omega) \\ m^{-\tau} + \text{infinite} \quad \text{aggregate} & \rho > \rho_c(\omega), \end{cases} \quad (1)$$

where ρ_c is given as $\rho_c(\omega) = \sqrt{\omega+1} - 1$. ρ_c and τ are shown to be independent of the spatial dimension d [16]. The tail of the mass distribution changes from exponential to an algebraic decay as ρ approaches ρ_c from below. As one further increases ρ beyond ρ_c, this asymptotic algebraic part of the critical distribution remains unchanged, but in addition an infinite aggregate forms. This means that all the additional mass $(\rho-\rho_c)L^d$ condenses onto a single site and does not disturb the background critical distribution. The $\omega = \infty$ case corresponds to ZRP with a constant chipping rate, and then there is no condensation transitions on regular lattices [5]. The critical exponent τ is same everywhere on the critical line $\rho_c(\omega)$. Recent studies showed that the existence of the condensation transitions depends on the

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spatial disorder [17], the symmetry of moving directions [18], the constraint of diffusion rate [19], and the underlying network structure [20].

In more general situations, as pointed out in Ref. [15], the diffusion rate of mass and the chipped mass will depend on the mass of a departure site. When the diffusion rate depends on mass as $D(m) \sim m^{-\alpha}$ with $\alpha > 0$ [19], the condensed phase disappeared in the thermodynamic limit. However finite systems undergo the condensation transitions at a certain critical density ρ_c which algebraically diverges with the system size L. On the other hand, the mass-dependent fragmentation or chipping is also important in general physical situation. For instance, in gelation phenomena, there is no reason that only one monomer at the end of a polymer is chipped regardless of the mass of polymers. It is very likely that more monomers can be chipped for more massive polymers. Hence it is natural to study the effect of mass-dependent fragmentation on the condensation phase transitions. In this paper, we investigate the condensation transitions of SCA models in which the fragmented mass in a chipping process depends on the whole mass of the departure site.

We consider SCA model with mass-dependent fragmentation in one dimension. In the model, the fragmented mass in the chipping process is given as $\delta m = Am^{\lambda}$. By chipping process with the rate ω, the mass δm randomly moves to the one of the nearest neighbors, $m_i \to m_i - \delta m_i$ and $m_{i\pm1} \to m_{i\pm1} + \delta m_i$. With the unit rate, mass m_i randomly diffuses to the one of the nearest neighbors, $m_i \to 0$ and $m_{i\pm1} \to m_{i\pm1} + m_i$.

Since the chipped mass is smaller than the whole mass of a site for $\lambda < 1$, the attractive interaction exists between masses on the same site. The $\lambda = 0$ case is just SCA model of Ref. [15]. The $\lambda = 1$ is a trivial point, but two extreme situations emerge according to the values of A. For $A = 0$ or 1, masses only diffuse without chipping so the complete condensation takes place. On the other hand, for $0 < A < 1$, the fragmented mass is proportional to the whole mass, which means that any aggregates cannot exit. Hence, the system is always in the fluid phase. The $\lambda = 1$ and $0 < A < 1$ case corresponds to the random fragmentation model in which $P(m)$ always exponentially decays [21]. On the other hand, for $\lambda < 0$, the complete condensation should always occur because the time needed to dissipate an aggregate, if exists, exponentially increases with the mass of an aggregate. Therefore, we focus on the range of $0 < \lambda < 1$.

We perform Monte Carlo simulations on a ring of size L. With random initial distributions of total mass ρL, the system is allowed to reach the steady states. We run simulations typically up to $t = 10^9$ time steps for size L up to 2048. In the steady states, we measure the single-site mass distribution $P(m)$ for various values of λ and ω. Fig.1 shows $P(m)$ of $\lambda = 0.1$ and 0.25 with $\rho = 20$ and $\omega = 0.1$ for L up to 2048. For both λ values, an aggregate with mass $(\rho - \rho_c)L$ forms and its probability decreases as $1/L$. ρ_c is a certain critical density above which an

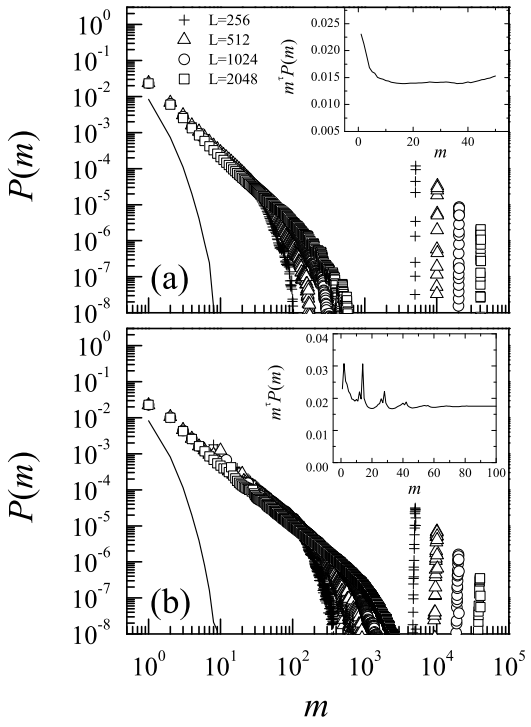

FIG. 1: The double logarithmic plots of $P(m)$ for $\lambda = 0.1$ (a) and 0.25 (b) with $\omega = 0.1$. Insets shows the scaling plot $m^{-\tau}P(m)$ with $\tau = 1.84$ (a) and 1.62 (b) for the data of $L = 2048$. In each panel, the solid line corresponds to $\rho = 0.01$ with $L = 2048$. Symbols are the data of $\rho = 20$ with L up to 2048.

aggregate forms. $P(m)$ shows that in finite systems, the condensation phase transitions occurs at finite ρ_c for $\lambda > 0$. For other values of ω, the condensation transitions also occur in finite systems. ρ_c increases with ω while τ is the same for all ω.

If there exists the condensation transitions in the thermodynamic limit, τ should satisfy $\tau > 2$. We measure the exponent τ from the scaling plot of $m^{\tau}P(m)$. For $L = 2048$ with $\omega = 0.1$, we estimate τ as 1.84(10) for $\lambda = 0.1$ and 1.62(10) for $\lambda = 0.25$. For other values of ω, we obtain the similar τ within errors. However, the finite size effect appears at very small mass even for large L, for example, at about $m = 100$ for $L = 2048$. Even though the precise measurement of τ is difficult due to the strong finite size effect, our estimates of τ are smaller than 2. It means that the condensation transitions take place only in the finite systems and disappear in the thermodynamic limit. In what follows, we discuss that no condensation transitions occurs in the thermodynamic limit.

In infinite systems with finite density (or $L \to \infty$), τ must be greater than 2 since the first moment $< m >$ is the density. Otherwise, the first moment diverges as $\ln L$ for $\tau = 2$ and L^{β} for $\tau < 2$ respectively. Hence

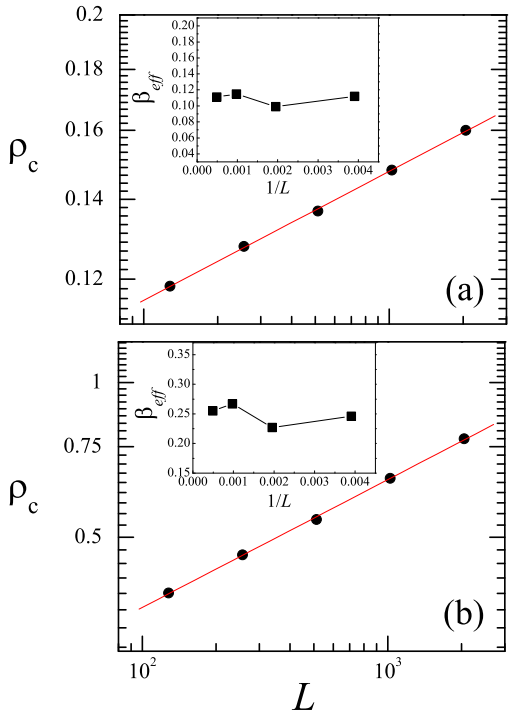

FIG. 2: The double logarithmic plots of $\rho_c(L)$ for $\lambda = 0.1$ (a) and 0.25 (b) with $\omega = 0.1$ and $\rho = 20$. The slope of line in each panel is $\beta = 0.1$ (a) and 0.25 (b).

$\tau < 2$ for $\lambda > 0$ contradict with the conservation of total density. Similar finite size effect with $\tau \leq 2$ has been observed in asymmetric CA [18] and SCA with mass-dependent diffusion rate [19]. ρ_c diverges with L as $\rho_c \sim \ln L$ for asymmetric CA and L^β for the mass-dependent diffusion rate respectively. In the thermodynamic limit or $L \to \infty$, no transitions occurs due to $\rho_c = \infty$ and a system is always in the fluid phase. As a result, $\tau < 2$ for $\lambda > 0$ implies that $\rho_c = \infty$ and the condensation transitions disappear in the thermodynamic limit. Only finite systems undergo the condensation transitions at a finite ρ_c.

To find the size dependence of ρ_c, we measure ρ_c for each L as follows. In the steady states, $P(m)$ scales as [16]

$$P(m) = m^{-\tau} f(m/L^\phi) + \frac{1}{L}\delta(m - (\rho - \rho_c)L) \ , \qquad (2)$$

where ϕ is the crossover exponent [16]. From the conservation of total masses, ρ_c is given as $\rho_c = \rho - \rho_\infty$, where ρ_∞ is the density of an aggregate. From the fact that the background distribution does not change for $\rho \geq \rho_c$, one can estimate ρ_c via $\int_1^{m_0} mP(m)dm$ in the condensed phase, where m_0 is the cut-off mass at which the background distribution terminates. Using this method, we confirmed that our estimate for ρ_c of the normal SCA model [15] converges to the the exact ρ_c as L increases (not shown). In Fig. 2, we plot ρ_c as a function of L for $\lambda = 0.1$ and 0.25 with $\omega = 0.1$, where we measure ρ_c using $P(m)$ for $\rho = 20$. Insets show the local slope β_{eff} defined as $\beta_{eff} = \ln[\rho(L)/\rho(L/b)]/\ln b$ with $b = 2$. As one can see, ρ_c diverges with L as L^β with $\beta = 0.11(3)$ for $\lambda = 0.1$ and $\beta = 0.25(5)$ for $\lambda = 0.25$. Since the fragmented masses increases for $\lambda \to 1$, ρ_c should diverge faster for larger λ. Hence we conclude that there are no condensation transitions in the thermodynamic limit for $0 < \lambda < 1$.

The scaling relation between three exponents, τ, ϕ and β is obtained by calculating $\rho_c = \int_1^{m_c} mP(m)dm$ for $\rho \geq \rho_c$, where m_c is the crossover mass over which $P(m)$ decays exponentially due to the finite size effect. The lower limit is the mass from which $P(m)$ begins to decay algebraically to the upper limit m_c. We set the lower limit of the integral to unit for convenience. Since m_c scales with L as $m_c \sim L^\phi$, ρ_c scales as $L^{\phi(2-\tau)}$. Hence β is given as $\beta = \phi(2 - \tau)$. From the normalization condition of $P(m)$, one obtains another scaling relation $\phi(\tau - 1) = 1$ [16]. Hence we have the two scaling relations,

$$\beta = \phi(2 - \tau) \ , \ \ \phi(\tau - 1) = 1 \ . \qquad (3)$$

From the two relations, τ is given as $\tau = (\beta + 2)/(\beta + 1)$. With the estimates of β, we obtain $\tau = 1.91$ for $\lambda = 0.1$ and 1.8 for $\lambda = 0.25$ respectively. Since the direct measurement of τ crucially depends on the choice of the scaling region, the estimate of τ is very subjective. On the other hand, ρ_c exhibits the nice scaling with L as shown in Fig. 2, the prediction of τ by the scaling relation should be more precise. Hence we predict $\tau = 1.91(3)$ for $\lambda = 0.1$ and 1.80(3) for $\lambda = 0.25$.

Recent studies showed that SCA models is well described by mean field theory [15, 16, 17, 18, 19]. Hence we expect that the present model is also well described by the mean field theory. To prove no transitions in the thermodynamic limit, we first assume the existence of the condensed phase, so that there is an aggregate. We also assume that all sites except the site occupied by an aggregate are occupied by mass ρ_c, which reflects the uniform background distribution with mass ρ_c. Then, for the existence of an aggregate in the steady states, the mass loss of an aggregate by chipping should be equal to the gain from the background. However the mass of an aggregate diverges with L, so the chipped mass of an aggregate also diverges as L^λ. On the other hand, the background distribution should not change in the condensed phase, the gain from the background is finite and independent of L. As a result, the loss diverges in the thermodynamic limit while the gain is finite. Therefore an aggregate cannot exist in the thermodynamic limit, which contradicts with the assumption of the existence of an aggregate. So the condensed phase cannot exist and the system is always in the fluid phase in the limit $L \to \infty$. Since the argument is based on the mean field, we expect no transitions in

any dimensions.

In summary, we investigate the condensation phase transitions of SCA model with a mass-dependent fragmentation in one dimension. We numerically find that condensation transitions take place only in finite systems. The critical density diverges with system size L as L^{β} because of $\tau < 2$ for $0 < \lambda < 1$. Hence in the thermodynamic limit of $L \to \infty$, no condensation transitions occur, so that a system is always in the fluid phase for $\lambda > 0$. Based on the mean field argument, we conclude that there are no condensation transitions in any dimensions.

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